HYDRAULIC TURBOMACHINES
Exercises 6 - Francis Turbines

Hydropower plant equipped with Francis turbines

La Grande III power plant is located in Northern Québec on La Grande river. LG III power plant features 12 units of 200 MW capacity each. Their main characteristics at the best efficiency point are listed in Table 1.

Table 1: LG III Francis main characteristics at BEP

Parameter	Definition
$E = 780\ J \cdot kg^{-1}$	Available specific energy
$Q = 275\ m^3 \cdot s^{-1}$	Discharge
$N = 112.5\ min^{-1}$	Rotating speed
$Z_{\bar{B}} = 175.6\ m$	Elevation of the tailrace free surface
$C_{\bar{I}} = 0.86\ m \cdot s^{-1}$	Flow velocity at the draft tube outlet
$D_{1e} = 5.484\ m$	Diameter at the leading edge - shroud intersection
$D_{\overline{1e}} = 5.240\ m$	Diameter at the trailing edge - shroud intersection
$D_0 = 6.680\ m$	Diameter at the guide vane axis
$B_0 = 1.397\ m$	Inlet channel height
$\eta_{elec} = 99.3\ \%$	Generator efficiency
$\eta_m = 98.8\ \%$	Mechanical efficiency
$\eta_q = 99.6\ \%$	Volumetric efficiency

1) Compute the torque T at the turbine-generator coupling.

$$P_{elec} = \eta_{elec}\ P$$

$$T = \frac{P}{\omega} = \frac{P_{elec}}{\eta_{elec}} \frac{60}{2\pi N} = 17.096\ MN \cdot m$$

2) Calculate η, the global efficiency of the turbine.

$$\eta = \frac{P}{P_h} = \frac{P}{\rho Q E} = 93.92\ \%$$

3) Neglecting the disc friction power losses, compute the discharge traversing through the turbine runner, Q_t , and the transformed specific energy, E_t .

$$Q_t = \eta_q Q = 273.90\ \ m^3 \cdot\ s^{-1}$$

$$\frac{P}{\eta_m} = P_m = P_t \quad if\ the\ disc\ friction\ power\ losses\ are\ neglected$$

$$P_t = \rho Q_t E_t \ \rightarrow\ E_t = \frac{P}{\rho Q_t \eta_m} = 744.42\ \ J \cdot\ kg^{-1}$$

4) Calculate the meridional component of the absolute flow velocity at the runner inlet and at the runner outlet, Cm_{1e} and $Cm_{\bar{1}e}$, respectively.

$$Cm_{1e} = \frac{Q_t}{A_1} = \frac{Q_t}{\pi D_{1e} B_0} = 11.38 \text{ m s}^{-1}$$

$$Cm_{\bar{1}e} = \frac{Q_t}{A_{\bar{1}}} = \frac{Q_t}{\frac{\pi}{4} D_{\bar{1}e}^2} = 12.70 \text{ m s}^{-1}$$

5) Calculate the blade angle at the shroud, $\beta_{\bar{1}e}$.

The blade angle at the runner outlet can be computed considering the outlet velocity triangle. Since the turbine is operating at BEP, $\alpha_{\bar{1}e} = 90°$ therefore $Cm_{\bar{1}e} = C_{\bar{1}e}$. By trigonometry:

$$\beta_{\bar{1}e} = \tan^{-1}\left(\frac{Cm_{\bar{1}e}}{U_{\bar{1}e}}\right) = \tan^{-1}\left(\frac{Cm_{1e}}{\omega \frac{D_{\bar{1}e}}{2}}\right) = 22.37°$$

6) Calculate the blade angle at the shroud, β_{1e}.

To compute β_{1e}, we need to define the components of the velocity triangle at the runner inlet. Since we are at BEP, $Cu_{\bar{1}e} = 0$, so to compute Cu_{1e} we can then use Euler equation:

$$E_t = U_{1e} Cu_{1e} - \cancel{k_{Cu\bar{1}e} U_{\bar{1}e} Cu_{\bar{1}e}} \rightarrow Cu_{1e} = \frac{E_t}{\omega \frac{D_{1e}}{2}} = 23.04 \text{ m s}^{-1}$$

Therefore, by trigonometry :

$$\beta_{1e} = \tan^{-1}\left(\frac{Cm_{1e}}{U_{1e} - Cu_{1e}}\right) = \tan^{-1}\left(\frac{Cm_{1e}}{\omega \frac{D_{1e}}{2} - Cu_{1e}}\right) = 50.85°$$

7) Estimate the guide vane opening, α_0, by assuming that it is equal to the absolute flow angle at the guide vane axis location. It can be assumed that the flow angular momentum is conserved between the guide vane and the runner inlet.

To compute the guide vane opening α_0, we need to define the components of the velocity triangle at the guide vanes, i.e. at the position with notation index $_0$.

The meridional component is simply retrieved by the discharge traversing radially the cylindrical section at the guide vanes position:

$$Cm_0 = \frac{Q_t}{A_0} = \frac{Q_t}{\pi D_0 B} = 9.34 \text{ m s}^{-1}$$

The tangential component can be retrieved considering that the flow angular momentum is conserved between the guide vanes and the runner inlet, i.e. that $\rho \cdot Cu \cdot R$ is conserved between these two sections:

$$\rho Cu_1 \frac{D_{1e}}{2} = \rho Cu_0 \frac{D_0}{2} \rightarrow Cu_0 = Cu_1 \frac{D_{1e}}{D_0} = 18.91 \text{ m s}^{-1}$$

Then, by trigonometry :

$$\alpha_0 = \tan^{-1}\left(\frac{Cm_0}{Cu_0}\right) = 26.29°$$

8) Sketch properly the inlet and outlet velocity diagrams. Use the grid provided in Figure 1.

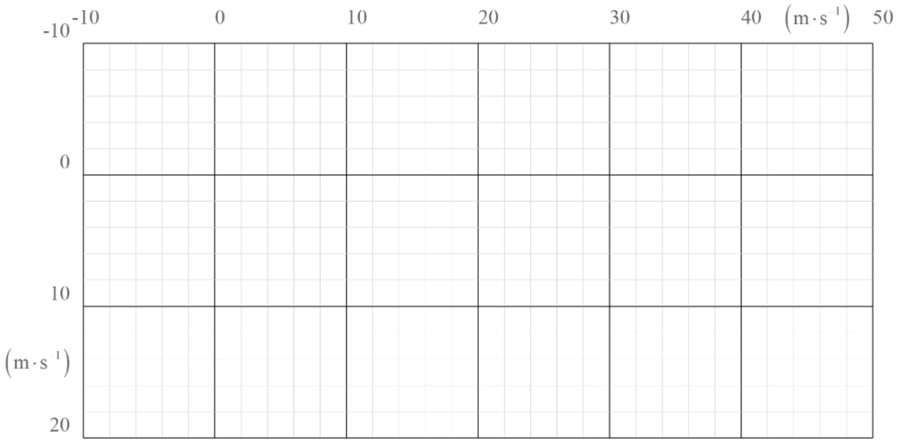

Figure 1: Velocity diagram at the turbine inlet (top) and outlet (bottom).

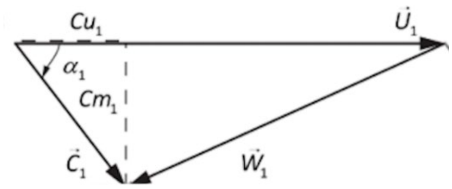

Figure 2: velocity triangle at the inlet

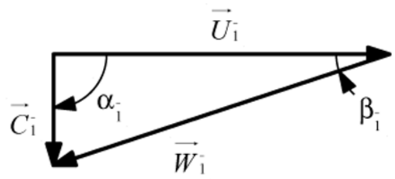

Figure 3: velocity triangle at the outlet

9) How to operate the machine with a lower discharge to decrease the power output, the head being constant?

The head being constant, the guide vane opening angle must be decreased

10) The machine is now operating at partial load. Illustrate qualitatively the new situation with the velocity triangle at the runner outlet.

Operating out of the BEP implies an outlet swirling flow, i.e. with a non-zero tangential component $Cu_{\bar{1}e} \neq 0$. *At partial load, this tangential component is such that* $\alpha_{\bar{1}e} < 90°$